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Remarks on experimental bounds on quantum gravity effects on fermions

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Abstract

Effects of space time geometry fluctuations on fermionic fields have recently been looked for in nuclear physics experiments, and were found to be much lower than predicted, at a phenomenological level, by loop quantum gravity. We show that possible corrections to the canonical structure in the semi classical regime may introduce important changes in the outcome of the theory, and may explain the observed mismatch with experiments.

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I. INTRODUCTION

It has only recently been realized that certain quantum gravity corrections to Maxwell and spinor field equations can lead to measurable effects. Tiny corrections in the propagation of photons or neutrinos accumulate through cosmological distances, giving rise to potentially observable effects [1] that would imply a breakdown of Lorentz invariance. In particular, for photons, this might take the form a birefringence effect [2], but, as indicated in [3], the observations of polarization in the visible and ultraviolet light from certain cosmological sources already imply an important upper bound on the effect. In the case of fermions, it has been shown in [4] that the breakdown could, in principle, be confirmed by means of extremely sensitive isotropy tests in nuclear systems [4]. The results obtained in [3] and [4], however, provide experimental bounds that seem to indicate that the Lorentz violation, if present, is far smaller than suggested by the theoretical predictions, pointing to an apparent discrepancy between theory and experiment. Seeking an explanation for this mismatch in the case of [4], we consider here the possibility that the canonical structure of the effective low energy theory gets corrections that vanish in the $\ell_P \to 0$ limit. This introduces additional terms in the field equations that may cancel some of the Lorentz violating effects, and therefore conciliate theory with experiment.

Let us start by recalling the action for a Majorana spinor in Minkowski space time

$$S = \int d^4x \left[i\bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{n\dot{\alpha}\alpha} \partial_n \xi_\alpha - \frac{m}{2}(\xi_\alpha \epsilon^{\alpha\beta} \xi_\beta - \bar{\xi}_{\dot{\alpha}} \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\xi}_{\dot{\beta}}) \right] \tag{1}$$

Here ${\sigma^0}_{\alpha\dot{\alpha}} = \bar{\sigma}^{0\dot{\alpha}\alpha} = \mathbb{I}$ and $-\bar{\sigma}^{j\dot{\alpha}\alpha} = {\sigma^j}_{\alpha\dot{\alpha}} =$ Pauli matrices. Spinor indices are raised and lowered using the second index of the antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\alpha\beta}$ ($\epsilon^{12} = \epsilon_{21} = 1$, and similarly for $\epsilon^{\dot{\alpha}\dot{\beta}}$.) (21)The equation of motion that follows from (1)

$$i\bar{\sigma}^{n\dot{\alpha}\alpha} \partial_n \xi_\alpha + m\epsilon^{\dot{\alpha}\dot{\beta}} \bar{\xi}_{\dot{\beta}} = 0 \tag{2}$$

gives the $\ell_p = 0$ limit of eq.(111) in [5] ((7) in [6],(1) in [4]). From (1) we obtain the field π^α conjugate to ξ_α

$$\pi^\alpha = i\bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{0\dot{\alpha}\alpha} \tag{3}$$

When performing a $3+1$ decomposition to write down the Hamiltonian, the $SL(2,)$ Lorentz symmetry is broken down to an $SU(2)$ rotation subgroup, under which dotted and undotted spinors are made equivalent by the $SU(2)$ invariant tensor ${\sigma^0}_{\alpha\dot{\alpha}}$ and its inverse $\bar{\sigma}^{0\dot{\alpha}\alpha}$. Defining ${{\sigma^k}_\alpha}^\beta = -{\sigma^0}_{\alpha\dot{\alpha}} \bar{\sigma}^{k\dot{\alpha}\beta}, k = 1, 2, 3$, the Hamiltonian obtained from (1) can be written as

$$H = \int d^3x \left[\pi^\beta {{\sigma^k}_\beta}^\alpha \partial_k \xi_\alpha + \frac{m}{2} \left(\xi_\beta \epsilon^{\beta\alpha} \xi_\alpha - \pi^\beta \epsilon_{\beta\alpha} \pi^\alpha \right) \right] \tag{4}$$

The canonical equal time anti-commutation relations

$$\{\xi_\alpha(x^0, \mathbf{x}), \pi^\beta(x^0, \mathbf{x}')\} = i{\delta_\alpha}^\beta \delta(\mathbf{x} - \mathbf{x}') \tag{5}$$

together with the Hamiltonian (4) allow us to recover the field equations (2) for the Majorana spinor

$$\begin{aligned} i\dot{\xi}_\gamma &= [\xi_\gamma, H] = \int d^3x' \left([\xi_\gamma, \pi^\beta {{\sigma^k}_\beta}^\alpha \partial_k \xi_\alpha] - \frac{m}{2} [\xi_\gamma, \pi^\beta \epsilon_{\beta\alpha} \pi^\alpha] \right) \\ &= i{{\sigma^k}_\gamma}^\alpha \partial_k \xi_\alpha - im\epsilon_{\gamma\beta} \pi^\beta \end{aligned} \tag{6}$$

The generalization of (4) to generic backgrounds gives the fermionic piece H_F of the fermion-gravity Hamiltonian, eq. (3) in [5]. In the quantum regime, operator products in H_F are regularized following Thiemann [7]. A coherent $|\mathcal{L},\xi\rangle$ state in the Hilbert space $\mathcal{H}_{grav}\otimes\mathcal{H}_{fermion}$ that approaches a flat space for distances $d\gg\mathcal{L}\gg\ell_P$, and a smooth fermion field living in it, is postulated in [5, 6]. $|\mathcal{L},\xi\rangle$ is assumed to be peaked around a flat metric and connection, the expectation value of the spatial metric operator q_{ab} behaving as $\langle\mathcal{L},\xi|q_{ab}|\mathcal{L},\xi\rangle=\delta_{ab}+\mathcal{O}(\ell_p/\mathcal{L})$, and also peaked around a fermion field configuration that varies slowly at the scale $\mathcal{L}$. A low energy effective Hamiltonian is then defined as the expectation value of the regularized H_F in this state. The result is eq.(109) in [5]:

$$\begin{aligned}H_{eff} &\equiv \langle\mathcal{L},\xi|H_F|\mathcal{L},\xi\rangle\\ &= \int d^3x\left[\pi^\beta{\sigma^k}_\beta{}^\alpha\hat{A}\partial_k\xi_\alpha+\frac{i}{2\mathcal{L}}\pi^\alpha\hat{C}\xi_\alpha+\frac{m}{2}\left(\xi_\beta\epsilon^{\beta\gamma}{O_\gamma}^\alpha\xi_\alpha-\pi^\beta{O_\beta}^\gamma\epsilon_{\gamma\alpha}\pi^\alpha\right)\right].\end{aligned} \tag{7}$$

Here $\pi^\alpha\equiv i\bar{\xi}_{\dot\alpha}\bar{\sigma}^{0\dot\alpha\alpha}$ (eq. (3)), ${O_\gamma}^\epsilon=\alpha{\delta_\gamma}^\epsilon-i\beta{\sigma^k}_\gamma{}^\epsilon\partial_k$ and

$$\begin{aligned}\hat{A} &= 1+\kappa_1\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon}+\kappa_2\left(\frac{\ell_P}{\mathcal{L}}\right)^{2+2\Upsilon}+\frac{\kappa_3}{2}\ell_P^2\nabla^2\\ \hat{C} &= \kappa_4\left(\frac{\ell_P}{\mathcal{L}}\right)^{\Upsilon}+\kappa_5\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+2\Upsilon}+\kappa_6\left(\frac{\ell_P}{\mathcal{L}}\right)^{2+3\Upsilon}+\frac{\kappa_7}{2}\left(\frac{\ell_P}{\mathcal{L}}\right)^{\Upsilon}{\ell_P}^2\nabla^2\\ \alpha &= 1+\kappa_8\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon}\qquad \beta/\ell_P=\frac{\kappa_9}{2}+\frac{\kappa_{11}}{2}\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon}\end{aligned} \tag{8}$$

Υ is a positive constant introduced in [5] to allow non integer powers of $(\ell_p/\mathcal{L})$ in the expansion of the expectation value of the connection, a possibility that was not considered in the previous works [4, 6], where $\Upsilon=0$. The dimensionless constants κ_j are expected to be of order unity. An explicit construction of $|\mathcal{L},\xi\rangle$ -which is lacking- would allow us to evaluate all these constants. Similar derivations can be found in [2] for Maxwell fields and [8] for scalar, Maxwell and fermion fields, with the following conceptual difference: the effective Hamiltonian is defined as the *partial* expectation value $H_{eff}\equiv\langle\mathcal{L}|H_F|\mathcal{L}\rangle$ in the gravity sector $\mathcal{H}_{grav}$. Since the regularized fermionic Hamiltonian is normal ordered [8] and $|\mathcal{L},\xi\rangle$ coherent in the fermionic sector, both results look formally equal. However, in Thieman's approach, (7) is an *operator* in $\mathcal{H}_{fermion}$. From (5) and (7) we obtain the field equation eq.(111) in [5] ((7) in [6],(1) in [4])

$$\left[i\partial_t-i\hat{A}\sigma^j\partial_j+\frac{\hat{C}}{2\mathcal{L}}\right]\xi+m\left(\alpha-i\beta\sigma^j\partial_j\right)i\sigma^2\xi^*=0, \tag{9}$$

which reproduces (2) in the limit $\ell_P\to 0$, and gives QG corrections up to order $(E\ell_P)^2$, E a characteristic energy scale for the fermion [4, 6].
Equation (9) with $\Upsilon=\kappa_4=0$, and keeping only leading order corrections (i.e., setting all $\kappa_j=0$ except for $j=1,5,9$), was used in [4] to obtain a modified Dirac equation depending on κ_1,κ_5 and κ_9. This modified Dirac equation violates Lorentz symmetry, and therefore gives the time evolution in a preferred frame, which is understood to be the CMB frame.

The equation actually follows from the lagrangian [4]

$$L_D = \frac{i}{2}\bar{\Psi}\gamma^a\partial_a\Psi - \frac{m}{2}\bar{\Psi}\Psi + \frac{i}{2}\kappa_1(m\ell_p)\bar{\Psi}\gamma_a(g^{ab} - W^aW^b)\partial_b\Psi$$
$$+ \frac{1}{8}\kappa_9(m\ell_p)\bar{\Psi}\epsilon_{abcd}W^a\gamma^b\gamma^c\partial^d\Psi - \frac{1}{4}\kappa_5(m\ell_p)mW_a\bar{\Psi}\gamma_5\gamma^a\Psi \quad (10)$$

if we set $W^a = (1,0,0,0)$. Therefore (10) generalizes the theory to other frames if W^a is replaced by the measured CMB frame's four-velocity ($|\vec{W}| \simeq 1.23 \times 10^{-3}c$ from the Earth.) Lorentz violation comes entirely from the fixed 4-vector W^a in (10). However, this violation is severely restricted by data on high precision tests of rotational symmetry in atomic and nuclear systems, which, as shown in [4], can be used to set the following stringent bounds on the constants κ_1, κ_5 and κ_9, expected in principle to be of order unity:

$$|\kappa_1| < 3 \times 10^{-5} \qquad |\kappa_9 + \kappa_5| < 4 \times 10^{-9} \quad (11)$$

The above bounds suggest that theory and experimental evidence will only agree if, after constructing the semiclassical states and fulfilling the details left over in the derivation of (9), one finds that $\kappa_1 = 0$ and $\kappa_9 = -\kappa_5$, i.e., the first Lorentz violating term in (10) is absent, and the other two appear with suitable coefficients. While cancellations of terms of the same order cannot be excluded, there appears to be no particular reason for a small κ_1, and its smallness is particularly intriguing [4]. In the next Section we suggest an alternative formulation for the *effective low energy* description of quantum gravity in the femion sector. It makes essential use of the possibility that the choice of appropriate canonical variables for the effective theory may require the inclusion of corrective terms in the anticommutation relations, in such a way one may recover agreement with the observational bounds in a more natural way. The effect of these terms on the phenomenological description of neutrino propagation and some low energy nuclear physics experiments are considered respectively in Sections 3 and 4. The last Section is devoted to a brief summary and conclusions.

II. AN EXTENSION OF THE FORMALISM

Looking for an alternative explanation, we allow non integer powers of $(\ell_p/\mathcal{L})$ by restoring Υ in the operators (8), and further consider the possibility that, in the effective theory, ξ and $i\bar{\xi}_{\dot{\alpha}}\bar{\sigma}^{0\dot{\alpha}\alpha}$ do not anticommute canonically. This possibility arises quite naturally in an effective lagrangian approach. If we replace $\pi^\alpha \to i\bar{\xi}_{\dot{\alpha}}\bar{\sigma}^{0\dot{\alpha}\alpha}$ back in the expectation value (7) we obtain the effective energy

$$\int d^3x \left[i\bar{\xi}_{\dot{\beta}}\bar{\sigma}^{0\dot{\beta}\beta}\sigma^k{}_\beta{}^\alpha\hat{A}\partial_k\xi_\alpha + \frac{i}{2\mathcal{L}}i\bar{\xi}_{\dot{\alpha}}\bar{\sigma}^{0\dot{\alpha}\alpha}\hat{C}\xi_\alpha + \frac{m}{2}\left(\xi_\beta\epsilon^{\beta\gamma}O_\gamma{}^\alpha\xi_\alpha - i\bar{\xi}_{\dot{\beta}}\bar{\sigma}^{0\dot{\beta}\beta}O_\beta{}^\gamma\epsilon_{\gamma\alpha}i\bar{\xi}_{\dot{\alpha}}\bar{\sigma}^{0\dot{\alpha}\alpha}\right)\right]$$
$$\equiv \int d^3x\, \mathcal{E}_{eff} \equiv E_{eff}. \quad (12)$$

Dropping for simplicity, as in [4], sub leading order corrections in (8) (in particular, terms containing Laplacians) yields $\mathcal{E}_{eff} = \mathcal{E}_{eff}(\xi, \bar{\xi}, \vec{\nabla}\xi, \vec{\nabla}\bar{\xi})$. There is, however, a large freedom in the construction of effective lagrangians $\mathcal{L}_{eff}$ leading to the energy density $\mathcal{E}_{eff}$. In fact, since $\mathcal{L}_{eff}$ is expected to be linear in $\dot{\xi}$, for any $f^\alpha(\xi, \bar{\xi}, \vec{\nabla}\xi, \vec{\nabla}\bar{\xi})$,

$$\mathcal{L}_{eff} = \{\dot{\xi}_\alpha f^\alpha(\xi, \bar{\xi}, \vec{\nabla}\xi, \vec{\nabla}\bar{\xi}) + cc\} - \mathcal{E}_{eff}(\xi, \bar{\xi}, \vec{\nabla}\xi, \vec{\nabla}\bar{\xi}) \quad (13)$$

will do the job, since we loose track of the conjugate field f^α in the energy density. Different choices of f^α will, however, give different physics. Back to the Hamiltonian formalism, the dynamical information is recovered through the equal time anti-commutation relations

$$\{\xi_\alpha, f^\beta(\bar{\xi})\} = i\delta_\alpha{}^\beta \delta(\mathbf{x}-\mathbf{x}'). \tag{14}$$

together with the effective energy (12). Equivalently, we may adopt (3) as a *definition* and use (7), keeping in mind that π and ξ *do not* anticommute canonically. In view of (14), the anticommutator (5) will pick up corrective terms, the simplest possible correction being

$$\{\xi_\alpha(x^0,\mathbf{x}), \pi^\beta(x^0,\mathbf{x}')\} = i\eta\delta_\alpha{}^\beta \delta(\mathbf{x}-\mathbf{x}') + \ell\sigma^j{}_\alpha{}^\beta \partial_j \delta(\mathbf{x}-\mathbf{x}'). \tag{15}$$

with

$$\ell = \ell_P\left(\kappa + \kappa'\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon}\right) \qquad \eta = 1 + \tilde{\kappa}\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon} + \tilde{\kappa}'\left(\frac{\ell_P}{\mathcal{L}}\right)^{2+2\Upsilon} \tag{16}$$

We remark that we may well avoid an effective lagrangian argumentation of (15). In the partial expectation value approach of ref [8], $H_{eff} = \langle \mathcal{L}|H_F|\mathcal{L}\rangle$, the possibility of allowing the deformation (15) of the canonical structure of the effective theory is quite natural, since the gravitational sector of the full Hilbert space affects even the notion of normal order of the matter fields [8]. In the alternative approach of refs [5, 6], where $H_{eff} = \langle \mathcal{L},\xi|H_F|\mathcal{L},\xi\rangle$, the perturbed dynamics from (15) would account for the departures of the time evolution of expectation values of fermion fields from their classical trajectory, departures that are well known to occur already at order $\hbar$ in the simplest quantum mechanical systems, even if the initial state is coherent and properly fine tuned [9] (This happens because an initial coherent state evolves into non coherent states. For illustrations in specific 1-D systems see e.g. [10].)

Equation (7) together with (15) and $i\partial_t \xi_\gamma(t,\mathbf{x}) = [\xi_\gamma(t,\mathbf{x}), H_{eff}]$ give a modified equation for the Weyl spinor. We first evaluate the nonzero commutators:

$$\begin{aligned}
\int d^3x' [\xi_\gamma, \pi^\beta \sigma^k{}_\beta{}^\alpha \hat{A} \partial_k \xi_\alpha] &= \int d^3x' [i\eta\delta_\gamma{}^\beta \delta(\mathbf{x}-\mathbf{x}') + \ell\sigma^j{}_\gamma{}^\beta \partial_j \delta(\mathbf{x}-\mathbf{x}')]\sigma^k{}_\beta{}^\alpha \hat{A}\partial'_k \xi_\alpha \\
&= i\eta\sigma^k{}_\gamma{}^\alpha \hat{A}\partial_k\xi_\alpha + \ell\sigma^j{}_\gamma{}^\beta \sigma^k{}_\beta{}^\alpha \hat{A}\partial_j\partial_k\xi_\alpha \\
&= i\eta\sigma^k{}_\gamma{}^\alpha \hat{A}\partial_k\xi_\alpha + \ell\hat{A}\nabla^2\xi_\gamma,
\end{aligned} \tag{17}$$

$$\begin{aligned}
\int d^3x' \left[\xi_\gamma, \frac{i}{2\mathcal{L}}\pi^\alpha \hat{C}'\xi_\alpha\right] &= \frac{i}{2\mathcal{L}}\int d^3x' [i\eta\delta_\gamma{}^\alpha \delta(\mathbf{x}-\mathbf{x}') + \ell\sigma^j{}_\gamma{}^\alpha \partial_j\delta(\mathbf{x}-\mathbf{x}')]\hat{C}'\xi_\alpha \\
&= -\frac{\eta}{2\mathcal{L}}\hat{C}\xi_\gamma + \frac{i\ell}{2\mathcal{L}}\sigma^j{}_\gamma{}^\alpha \partial_j \hat{C}\xi_\alpha,
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
&\int d^3x' [\xi_\gamma, -\frac{m}{2}\pi^\beta O'_\beta{}^\omega \epsilon_{\omega\alpha}\pi^\alpha] = \\
&\qquad -\frac{m}{2}\int d^3x' (\{\xi_\gamma, \pi^\beta\} O'_\beta{}^\omega \epsilon_{\omega\alpha}\pi^\alpha - \pi^\beta O'_\beta{}^\omega \epsilon_{\omega\alpha}\{\xi_\gamma,\pi^\alpha\}) = \\
&\qquad - i\eta m\alpha\epsilon_{\gamma\alpha}\pi^\alpha - m\ell\alpha\sigma^j{}_\gamma{}^\beta \epsilon_{\beta\alpha}\partial_j\pi^\alpha - \eta m\beta\sigma^k{}_\gamma{}^\omega \epsilon_{\omega\alpha}\partial_k\pi^\alpha - m\ell\epsilon_{\gamma\alpha}\nabla^2\pi^\alpha
\end{aligned} \tag{19}$$

(here we have used $(\sigma^{(k}\sigma^{j)})_\alpha{}^\beta = \delta^{jk}\delta_\alpha{}^\beta$ and $(\sigma^{(k}\epsilon\sigma^{j)})_{\alpha\beta} = -\epsilon_{a\beta}\delta^{kj}$ and $\sigma^k{}_\alpha{}^\omega\epsilon_{\omega\beta} = \sigma^k{}_\beta{}^\omega\epsilon_{\omega\alpha}$.) Collecting terms and suppressing spinor indices we arrive at the following equation:

$$\left[i\partial_t - i\eta\hat{A}\sigma^j\partial_j + \frac{\hat{C}\eta}{2\mathcal{L}}\right]\xi + m\eta\left(\alpha - i\beta\sigma^j\partial_j\right)i\sigma^2\xi^* =$$
$$\ell\left[\hat{A}\nabla^2\xi + \frac{i}{2\mathcal{L}}\hat{C}\sigma^j\partial_j\xi + m(i\alpha\sigma^j\partial_j + \beta\nabla^2)(i\sigma^2)\xi^*\right] \quad (20)$$

Setting $\ell = 0, \eta = 1$ above we recover (9). Note that the operators on the right hand side of (20) are of the same type as those on its left hand side (with the exception of higher order corrections like $\nabla\nabla\xi$ in $\hat{A}\nabla\xi$.) Then there is the possibility that an explicit evaluation of the κ constants in the effective theory will show that the Lorentz symmetry breaking terms on the left and right hand sides of the equation cancel each other. In other words, the apparent symmetry breaking caused by the regularization, normal ordering and by further taking the partial expectation value of H_F in the gravitational sector, is absorbed by a proper identification of the conjugate variables in the low energy regime. In fact, Lorentz symmetry can only be partially restored, as seen by matching equal powers of $(\ell_P/\mathcal{L})$ in (20) to show that (15) and (16) amount to the following redefinitions of $\hat{A}, \hat{C}, \alpha$ and β in (9):

$$\begin{aligned}
\hat{A}' &= 1 + \left(\kappa_1 + \frac{\kappa\kappa_4}{2} + \tilde{\kappa}\right)\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon} + \left(\kappa_2 + \frac{\kappa\kappa_5}{2} + \frac{\kappa'\kappa_4}{2} + \tilde{\kappa}\kappa_1 + \tilde{\kappa}'\right)\left(\frac{\ell_P}{\mathcal{L}}\right)^{2+2\Upsilon} + \frac{\kappa_3}{2}\ell_P^2\nabla^2 \\
\hat{C}' &= \kappa_4\left(\frac{\ell_P}{\mathcal{L}}\right)^{\Upsilon} + (\kappa_5 + \kappa_4\tilde{\kappa})\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+2\Upsilon} + (\kappa_6 + \kappa_5\tilde{\kappa} + \kappa_4\tilde{\kappa}')\left(\frac{\ell_P}{\mathcal{L}}\right)^{2+3\Upsilon} \\
&\quad + \left[\left(\frac{\kappa_7}{2} - 2(\kappa\kappa_1 + \kappa')\right)\left(\frac{\ell_P}{\mathcal{L}}\right)^{\Upsilon}\ell_P{}^2 - 2\mathcal{L}\kappa\ell_P\right]\nabla^2 \qquad (21) \\
\alpha' &= 1 + (\kappa_8 + \tilde{\kappa})\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon} \qquad \beta'/\ell_P = \left(\frac{\kappa_9}{2} + \kappa\right) + \left(\frac{\kappa_{11}}{2} + \kappa\kappa_8 + \frac{\tilde{\kappa}\kappa_9}{2} + \kappa'\right)\left(\frac{\ell_P}{\mathcal{L}}\right)^{1+\Upsilon}
\end{aligned}$$

Interestingly enough, this potential restoration of Lorentz symmetry, if only partial, is enough to solve the contradiction found in [4].

III. QUANTUM GRAVITY CORRECTIONS TO NEUTRINO PROPAGATION

We analyze the consequences of the modifications introduced by (15) in (20) in the results in [6], where the propagation of neutrinos from GRB is considered. For these fermions, $p \sim 10^5 GeV$ and $m \sim 10^{-9} GeV$, thus $m/p \sim p\ell_P \sim 10^{-14} \equiv \epsilon$. Assuming $\Upsilon \simeq 0$, $\mathcal{L} \sim 1/p$, and keeping terms up to order $\epsilon^{2+3\Upsilon}$, we can see that all terms in (21) should be kept, except the $(\frac{\ell_P}{\mathcal{L}})^{1+\Upsilon}$ piece of β'/ℓ_P. This is so because $\partial_t \sim \nabla \sim \frac{1}{\mathcal{L}} \sim p$, whereas $m = \epsilon p$. Note that the laplacian term in $\hat{C}$, which is order ϵ^2, has picked up a κ correction of order ϵ. This is the most important change introduced by the correction (15). It implies helicity dependent effects of order $p\ell_P$ whenever $\kappa \neq 0$, as opposed to the predicted effects of order $(p\ell_P)^2$ in [6] (this is seen by noting that the term $\pm\kappa_7(p\ell_p)^2/2$ of equation (7) in [6] gets replaced by $\mp 2\kappa(p\ell_p)$.) Also, the replacements $\kappa_1 \to \kappa_1 + \kappa\kappa_4/2 + \tilde{\kappa}$, etc, may lead to cancellations that switch off some of the predicted QG effects on neutrinos. None of these predictions can be checked using currently available experimental data.

IV. QUANTUM GRAVITY CORRECTIONS IN NUCLEAR PHYSICS EXPERIMENTS

In [4], (10) is applied to non relativistic nucleons. The weave scale $\mathcal{L}$ is set equal to $1/m$, κ_4 is set equal to zero, and only leading order corrections are kept. The results are the bounds (11). We will instead assume Υ to be a small (less than one) positive number and use (21) keeping terms up to order $1+\Upsilon$. Equation (21) reduces to

$$\begin{aligned} \hat{A}' &= 1 + (\kappa_1 + \tilde{\kappa})\,(m\ell_P)^{1+\Upsilon} \qquad \hat{C}' = \kappa_5 (m\ell_P)^{1+2\Upsilon} \\ \alpha' &= 1 + (\kappa_8 + \tilde{\kappa})(m\ell_P)^{1+\Upsilon} \qquad \beta'/\ell_P = \left(\frac{\kappa_9}{2} + \kappa\right) \end{aligned} \tag{22}$$

and the bounds predicted in [4] now read

$$\left|(\kappa_1 + \tilde{\kappa})\,(m\ell_P)^{\Upsilon}\right| < 3 \times 10^{-5} \qquad \left|\kappa_9 + 2\kappa + \kappa_5 (m\ell_P)^{2\Upsilon}\right| < 4 \times 10^{-9} \tag{23}$$

(the κ_5 term should be dropped if $\Upsilon \neq 0$.) These bounds can safely be satisfied for κ's of order unity, unless $\Upsilon = \kappa = \tilde{\kappa} = 0$. This shows that admitting an expansion in non-integer powers of $(\ell_P/\mathcal{L})$, as in [5], and the corrections (5) to the anti-commutator in the semiclassical regime, the contradictions found in [4] can be solved without questioning the basic framework of loop quantum gravity.

V. CONCLUSIONS

The efforts directed at the construction of a quantum theory of gravity based on a canonical formalism have led in recent years to a formulation that appears quite promising, but that is so far incomplete. In particular, a definite procedure connecting the theory with the low energy regime is still lacking, and one has to resort to plausibility arguments as to the final form of the theory in order to make predictions regarding the observable consequences of the theory in that domain. Nevertheless, using as a guide essentially heuristic notions, a small number of possible phenomena where the quantum gravity effects might be sought, together with estimations of their magnitude, have been indicated in the recent literature. A common feature in all cases, however, is that the observable data, rather than indicating their presence, seems to impose severe upper bounds on their possible existence. But, again a common feature of these effects is they all violate in one form or another (local) Lorentz invariance. Since it is not at all clear that the complete theory should necessarily lead to such violation, in this Rapid Communication we have suggested a way in which, in the fermion sector, a modification of the *effective* canonical structure in the low energy regime might naturally lead to a better agreement with observational data and compatibility with Lorentz invariance.

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